Fen-Edebiyat Fakültesi	FINAL EXAM	
Name, Surname:	Department:	GRADE
Student No:	Course: Diff Equs	
Signature:	Exam Date: 10/01/2019	

Each problem is worth 20 points. Duration is 80 minutes.

Lap. Tran.: (A) $\mathscr{L}(e^{at}) = \dfrac{1}{s-a} = e^{at}$, (B) $\mathscr{L}(\sin(at)) = \dfrac{a}{s^2+a^2}$, (C) $\mathscr{L}(\cos(at)) = \dfrac{s}{s^2+a^2}$, (D) $\mathscr{L}(t^n) = \dfrac{n!}{s^{n+1}}$, (E) $\mathscr{L}(u_c(t)) = \dfrac{e^{-cs}}{s}$, (F) $\mathscr{L}(u_c(t)f(t-c)) = e^{-cs}\mathscr{L}\big(f(t)\big)$, (G) $\mathscr{L}(f''(t)) = s^2\mathscr{L}(f(t)) - sf(0) - f'(0)$,

1. Solve $y'' - 4y = f(t), \quad y(0) = 0 \quad y'(0) = 0, \quad f(t) = \begin{cases} 0, & 0 \le t \le 5 \\ 8, & t \ge 5 \end{cases}$

Solution: Note $f(t) = 8u_5(t)$. Take Laplace transform of both sides to get $(s^2-4)Y(s) = 8\frac{e^{-5s}}{s}$. Find the inverse laplace transform of $\dfrac{8}{s(s^2-4)} = \dfrac{-2}{s} + \dfrac{2}{s-2} - \dfrac{1}{s+2}$ which is $h(t) = -2 + 2e^{2t} - e^{-2t}$. $y(t) = L^{-1}Y(s) = u_5(t)h(t-5)$.

2. Solve $\dfrac{dy}{dx} = xe^{x+y}$, $y(0) = \ln 2$.

Solution: $\displaystyle\int e^{-y}dy = \int xe^x dx \implies -e^{-y} = xe^x - e^x + C$. The initial condition gives $-e^{-\ln 2} = -1 + C \implies C = 1 - e^{\ln 1/2} = 1/2$. The solution is $e^{-y} + e^x(x-1) + 1/2 = 0$.

3. Solve $y' - 2y = e^{2t}, \quad y(0) = 2$.

Solution: Integrating factor is $\mu(t) = e^{-2t}$. $\dfrac{d}{dt}\left(ye^{-2t}\right) = e^{2t}e^{-2t} = 1 \implies ye^{-2t} = t + C \implies y = te^{2t} + Ce^{2t}$

$y(0) = 2 \implies C = 2$. The solution is $y = te^{2t} + 2e^{2t}$.

4. Solve $x^2y'' + 4xy' + 2y = 0$ with initial conditions $y(1) = 1$, $y'(1) = 4$.

Solution: $y = x^r \implies r(r-1)x^r + 4rx^r + 2x^r = 0 \implies r^2 + 3r + 2 = 0 \implies r = -2 \quad r = -1$ The solution is $y = c_1x^{-2} + c_2x^{-1}$. Initial conditions give $y = -5x^{-2} + 6x^{-1}$.

5. Find the inverse Laplace transform of $F(s) = \dfrac{8s^2 - 4s + 12}{s\left(s^2+4\right)}$.

Solution: $\dfrac{8s^2-4s+12}{s\left(s^2+4\right)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+4}$. We find $A = 3$, $B = 5$, $C = -4$. Laplace transform table gives $f(t) = 3 + 5\cos 2t - 2\sin 2t$.